Adaptive Optics Performance Model for Optical Interferometry

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ABSTRACT

We argue that adding large apertures with adaptive optics to an optical interferometer improves the performance of the interferometer in two ways: it improves the signal to noise of bright, low-visibility fringes and it also improves the sensitivity of the interferometer. A simple model is presented to support this conclusion.

Keywords: Optical Interferometry, Adaptive Optics

1. INTRODUCTION

It has been argued, see,[1] that adding large telescopes with adaptive optics (AO) to an optical interferometer (OI) provides no benefit because both techniques have similar limiting magnitudes. Even if there is no gain in sensitivity, adaptive optics will clearly enable observations of sources with smaller visibility amplitudes. Any observation more exciting than the diameter of a single star or a binary star orbit, will require fringe visibility measurements down to the order $V \approx 0.1$. Since the signal-to-noise-ratio (SNR) is proportional to $V\sqrt{N}$, where N is the number of photon per coherence volume, it is necessary to have apertures with $D/r_0 \approx 30$, assuming that $3r_0$ can be used for unresolved sources without AO.

In practice optical interferometers have low throughput likely due to the long and complex optical trains that transport the star light from the collecting aperture to the beam combiner. By putting the AO where it does not suffer from these losses, an order of magnitude increase in sensitivity may be possible. Since the optimum coherence volumes are not exactly the same for these two techniques there may be an additional increase in the sensitivity.

A third potential advantage of AO is that the large aperture averages over the high-frequency delay variations resulting in more slowly varying phases. Spatial filtering (e.g. single mode fibers) can improve the spatial coherence improving the stability of the amplitude measurements. For bright objects this improved phase stability provides an improvement in the temporal coherence further improving on what is obtained with spatial filtering.

We have started an experimental program to evaluate the effect of combining AO with OI.[2] In parallel and in support of this experimental program, we started an analytic effort to quantify the arguments presented above. This paper presents the initial sensitivity calculations from that effort. The next section describes the model containing the functional dependency of the most important parameters. Section 3 optimizes the performance of the AO system which is combined with an interferometer model in Section 4.

2. ADAPTIVE OPTICS MODEL

We consider an optical interferometer with two apertures, each of diameter D. For the adaptive optics we use sub-apertures of diameter A and a servo time constant t_s. We use a standard, Kolomogorov representation of the atmosphere with r_0 being the coherence length defined by Fried[3]

$$\sigma^2(B) = 6.88(B/r_0)^{5/3} \tag{1}$$

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where $\sigma^2(B)$ is the structure function, the variance of the phase difference (in radians) for two points separated by a distance B.

The corrected wavefront will not be perfectly flat but rather have errors due to three contributions: a residual tilt error due to the imperfect measurement of the tilt within each subaperture, higher order distortion within each subaperture that cannot be removed simply by tilting and pistoning each subaperture, and the wavefront error due to the evolution of the wavefront between the time of the measurement and the time the correction is performed. We will assume that these errors are independent and add their variances.

This model is not perfect. There are correlations between the errors. The model also lacks any scintillation. We argue that scintillation is unimportant for although it may affect the performance of the servo, areas in the wavefront contributing few photons to the wavefront sensor do not need to be corrected since they also contribute few photons to the interferometer. There is a second order effect since even a perfectly flat wavefront will not produce a perfect PSF in the presence of scintillation, but the prevailing viewpoint is that this is not a significant limitation to current adaptive optics system.

Residual tilt error. Consider a circular aperture of diameter A centered at the origin of the x, y plane. A tilt of θ radians along the X-axis produces a delay error across the aperture equal to $x\theta$. The mean delay is zero. The variance is given by

$$\sigma_d^2 = (\theta A)^2 \frac{1}{4\pi} \int\int x^2 dx dy = \left[\frac{\theta A}{4}\right]^2 \tag{2}$$

where the integral is over the unit circle. Assume there are N detected photons from a coherence volume from each aperture available for the adaptive optics. The phase error is given by

$$\sigma_\phi^2 = 2\left(\frac{2\pi}{\lambda}\right)^2 \left(\frac{A}{4}\right)^2 \left[\frac{(\lambda/A)^2}{N(A/r_0)^2(t_s/t_0)}\right] = \left(\frac{\pi^2}{N}\right)\left(\frac{A}{r_0}\right)^{-2}\left(\frac{t_s}{t_0}\right)^{-1} \tag{3}$$

The leading 2 accounts for the two dimensions of tip/tilt. The next term converts a path length error into radians and the second term converts angle into path length.

Higher order distortion. Because the adaptive optics is taking out the low frequency variations, the variance of the high-order distortion for the entire aperture is the same as the variance of a single sub-aperture. Noll[4] calculates these partially corrected wavefront variances. For Piston, tip and tilt completely removed, the relevant variance is Δ_3 from Noll's Table 4

$$\sigma^2 = 0.134\left(\frac{A}{r_0}\right)^{5/3} = \alpha\left(\frac{A}{r_0}\right)^{5/3} \tag{4}$$

Wavefront evolution. The wavefront sensor outputs a wavefront corresponding to its shape at the middle of the integration. This is always on the order of t_s in the past. By the time the correction is applied to the wavefront, the wavefront has evolved. Piston has no affect on the AO system so that evolution should not be included. Also, the high order variations are not being corrected; if they evolve into different high-order aberrations, there will be a different wavefront, but no change in the mean variance. It is only the evolution of tip/tilt that matters. For the sake of having a closed-form expression, we replace the tip/tilt power spectra with the power spectra corresponding to the phase difference between two points separated by A. This form has the correct total power but somewhat overestimates the high frequency contribution, a slightly conservative assumption.

$$\Phi(f)^2 = 0.0186 t_0^{-5/3} f^{-8/3} \quad \text{for} \quad f > f_0 \tag{5}$$

$$= 0.0186 t_0^{-5/3} f_0^2 f^{-2/3} \quad \text{for} \quad f < f_0 \tag{6}$$

$$f_0 = \frac{0.062 r_0}{A t_0} \tag{7}$$

where t_0 is the atmospheric coherence time with the definition advocated by Buscher.[5] The effect of a first-order servo with time constant t_s on the tip/tilt power can be approximated by multiplying the power spectrum by

$(ft_s)^2$ for frequencies less than $1/t_s$. For $f_0 t_s < 1$, the power that is not corrected by the angle tracking servo is the integral of this power spectrum and accounts for the time evolution term.

$$\sigma_E^2 = 0.067\left(\frac{t_s}{t_0}\right)^{5/3} - 0.0189\left(\frac{r_0}{A}\right)^{1/3}\left(\frac{t_s}{t_0}\right)^2 \tag{8}$$

Adding these together gives

$$\sigma^2 = \left(\frac{\pi^2}{N}\right)\left(\frac{A}{r_0}\right)^{-2}\left(\frac{t_s}{t_0}\right)^{-1} + 0.134\left(\frac{A}{r_0}\right)^{5/3} + 0.067\left(\frac{t_s}{t_0}\right)^{5/3} - 0.0189\left(\frac{A}{r_0}\right)^{-1/3}\left(\frac{t_s}{t_0}\right)^2 \tag{9}$$

3. OPTIMIZATION

The observing conditions are described by the coherence length, r_0, the coherence time t_0 and the brightness of the star. We parameterize the star brightness as N, the number of detected photons per coherence volume. Our job is to choose the subaperture size, A, and the integration time, t_s to minimize the phase variance of the wavefront. This is accomplished with two partial derivatives. Writing $a = A/r_0$, $t = t_s/t_0$, $\alpha = 0.134$ and $\beta = 0.067$, $\gamma = 0.0189$, we have

$$\sigma^2 = \frac{\pi^2}{a^2 t N} + \alpha a^{5/3} + \beta t^{5/3} - \gamma t^2 a^{-1/3} \tag{10}$$

$$\frac{\partial \sigma^2}{\partial t} = \frac{-\pi^2}{N a^2 t^2} + (5/3)\beta t^{2/3} - 2\gamma a^{-1/3} t \tag{11}$$

$$\frac{\partial \sigma^2}{\partial a} = \frac{-2\pi^2}{N a^3 t} + (5/3)\alpha a^{2/3} + (1/3)\gamma a^{-4/3} t^2 \tag{12}$$

Setting these partials to zero gives two equations with two unknowns

$$\frac{\pi^2}{N} = (5/3)\beta a^2 t^{8/3} - 2\gamma a^{5/3} t^3 \tag{13}$$

$$\frac{2\pi^2}{N} = (5/3)\alpha a^{11/3} t + (1/3)\gamma a^{5/3} t^3 \tag{14}$$

which can be solved for t and a. First eliminate the constant term

$$(10/3)\beta a^2 t^{8/3} = (5/3)\alpha a^{11/3} t + (13/3)\gamma a^{5/3} t^3 \tag{15}$$

$$10\beta = 5\alpha\left(\frac{a}{t}\right)^{5/3} + 13\gamma\left(\frac{a}{t}\right)^{-1/3} \tag{16}$$

Which has the solution

$$\rho = \frac{a}{t} = 0.73076 \tag{17}$$

then eliminate the final term

$$\frac{13\pi^2}{N} = (5/3)\beta a^2 t^{8/3} + 10\alpha a^{11/3} t \tag{18}$$

$$= (5/3)\beta a^{14/3}\rho^{-8/3} + 10\alpha a^{14/3}/\rho \tag{19}$$

$$a^{14/3} = \frac{39\pi^2}{5N\left[\beta\rho^{-8/3} + 6\alpha/\rho\right]} \tag{20}$$

$$a = 2.416 N^{-3/14} \tag{21}$$

The minimum variance occurs for this choices of a and ρ

$$\sigma^2 = \frac{\pi^2 \rho}{N a^3} + a^{5/3}\left(\alpha + \beta\rho^{-5/3} - \gamma\rho^{-2}\right) \tag{22}$$

$$\sigma^2 = 1.432 N^{-5/14}. \tag{23}$$

The results are shown in Figure 1 where the variance is plotted as a function of $N = \pi\eta_A F r_0^2 t_0/4 = \pi^2/\beta$, the number of photons per coherence volume delivered to the AO system. The optimum aperture and integration time are shown in Figure 2. Adaptive optics can be useful with only a few photons per coherence volume.

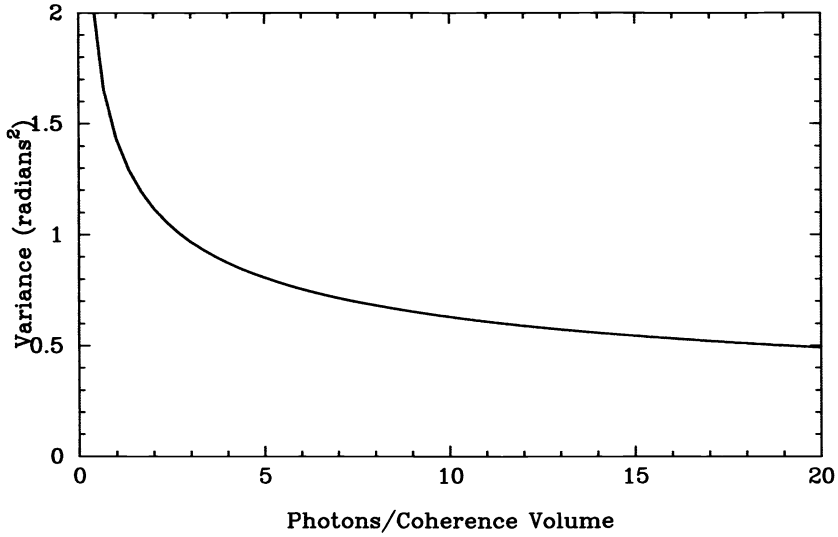

Figure 1. Performance of an AO system in terms of the variance of the corrected wavefront in terms of the number of photons per coherence patch delivered to the sensor.

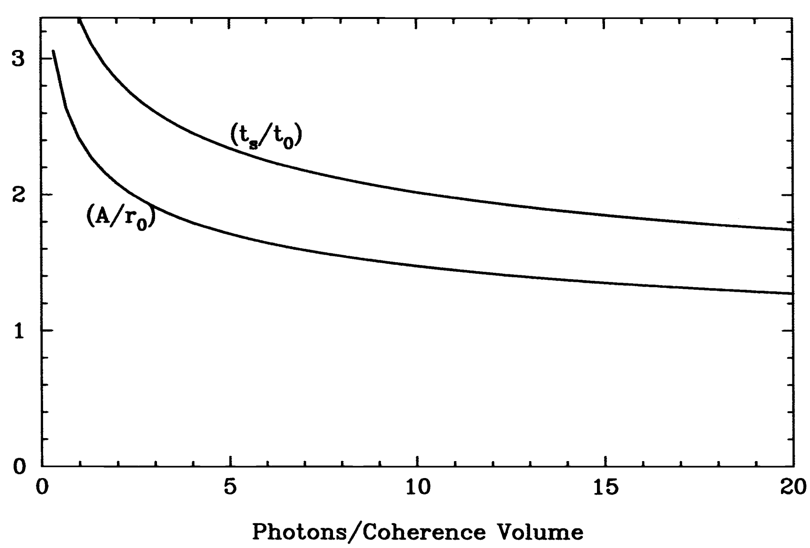

Figure 2. Best performance of a visible light adaptive optics system as a function of the number of photons per coherence patch delivered to the AO system. The line show A/r_0, the sub-aperture size in terms of the coherence length and t_s/t_0, the servo time constant in terms of the coherence time.

4. INTERFEROMETRY MODEL

To keep this model reasonably uncoupled from assumptions about the source and the instrument, we assume that the interferometry and the adaptive optics work off the same bandpass. Also, we assume the adaptive optics system will have higher throughput since it can be located at the telescope while the interferometry must be positioned at the end of a long optical train. A total of N photons is collected by each aperture. A fraction f will be selected for interferometry and $1 - f$ for the adaptive optics. We will assume efficiencies of $\eta_A = 0.5$ for the adaptive optics and $\eta_I = 0.2$ for interferometry. The number of photons per coherence volume available for the AO is

$$N_A = (1 - f)\eta_A N. \tag{24}$$

The number of photons per coherence volume for interferometry is

$$N_I = f\eta_I N. \tag{25}$$

The wavefront variance is given by Equation 10

$$\sigma^2 = \frac{\pi^2}{a^2 t N_A} + \alpha a^{5/3} + \beta t^{5/3} - \gamma t^2 a^{-1/3} \tag{26}$$

where a and t are the subaperture size and integration time for the adaptive optics, scaled by the coherence length and time, respectively. Finally, the square of the fringe signal to noise is

$$S = 2N_I (D/r_0)^2 e^{-2\sigma^2}. \tag{27}$$

This assumes photon-noise-limited observations and perfect spatial filtering giving a system visibility of unity. The coherence time for fringe detection has no effect on the specification of the AO so we ignore it here. We maximized S by varying a, t, and f. For comparison, we also performed the calculation for a system with only til/tilt corrections. This was done with the same equations but with D set equal to A.

Signal to noise as a function of aperture is shown in 3. This example is for 5 photons per coherence volume but the results are similar for other brightnesses. The lower line, for tip/tilt correction, gives the classic result of a maximum in the range of $D/r_0 = 3$. The AO result – a quadratic improvement with increasing aperture – may seem surprising but should be obvious. Increasing the aperture keeping the subaperture diameter constant has no affect on the wavefront variance but the number of photons available for fringe detection grows as the area of the telescope.

In Figure 4, we show the number of photons per aperture requried to obtain a fringe signal to noise of 1. With adaptive optics, larger apertures allow the observation of fainter stars.

5. SUMMARY

This paper presents an analytic model for assessing the performance of an optical interferometer fitted with adaptive optics. The performance of the interferometer, in terms of limiting magnitude, continues to improve with increasing aperture. These results are preliminary; clearly the model can be improved by including parametrization based on both detailed numerical analysis and experimental values. We are also planning an integrated model of all subsystems in order to better account for the interactions not captured by the present model.

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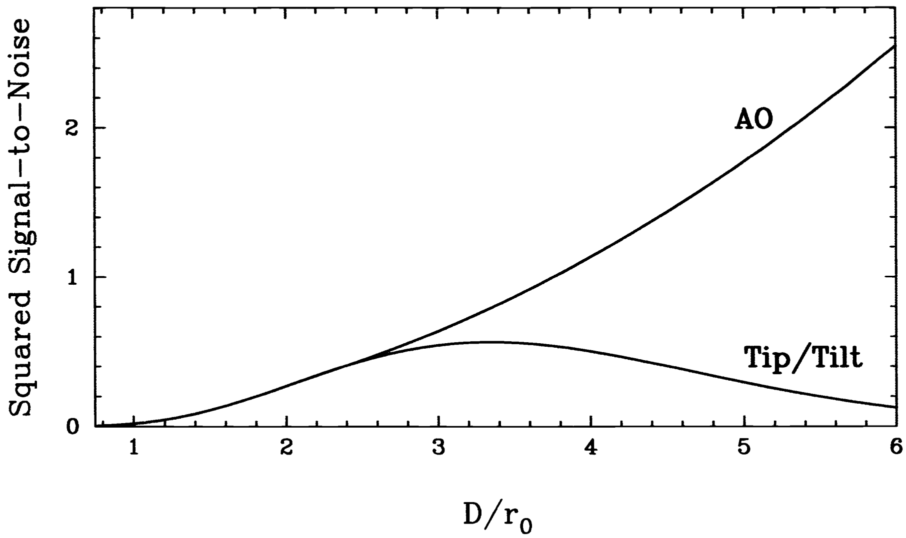

Figure 3. Comparison of the fringe signal to noise for telescopes with adaptive optics and with only tip/tilt corrections. For a system with AO, the SNR does not go through a maximum but continues to increase proportional to the collecting area.

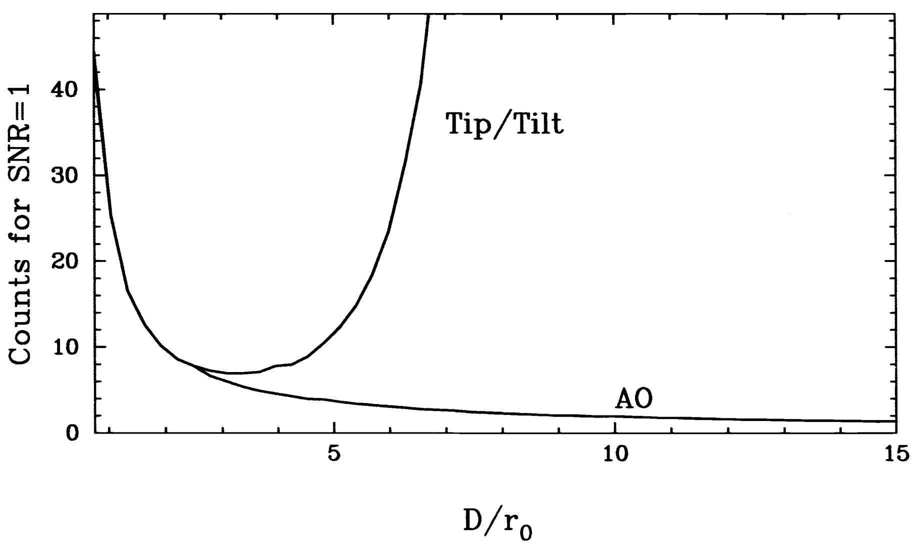

Figure 4. Comparison of the limiting sensitivity of optical interferometers with adaptive optics and tip/tilt corrections. As the diameter of the telescoeps is increased, the system with AO continues to gain sensitivity (although slowly) while the tip/tilt only system passes through a maximum in sensitivity.

REFERENCES

1. J. E. Baldwin and C. A. Haniff, "The application of interferometry to optical astronomical imaging," *Phil. Trans. R. Soc. Lond. A* **360**, pp. 969–986, 2002.
2. S. R. Restaino, D. M. Payne, D. Wick, and G. C. Gilbreath, "Experimental results from the use of a two deformable mirror system for optical interferometry," *SPIE* **5491**, 2004.
3. D. L. Fried *JOSA* **56**, p. 1372, 1966.
4. R. J. Noll *JOSA* **66**, p. 3, 1976.
5. D. F. Buscher, J. T. Armstrong, C. A. Hummel, A. Quirrenbach, D. Mozurkewich, K. J. Johnston, C. S. Denison, M. M. Colavita, and M. Shao, "Interferometric seeing measurements on mt. wilson: power spectra and outer scales," *Applied Optics* **34**, pp. 1081–1096, 1995.